

Interpreting CMB Anisotropy Observations: Trying to Tell the Truth with Statistics

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Abstract. A conflict has been reported between the baryon density inferred from deuterium observations and that found from recent CMB observations by BOOMERanG and MAXIMA. Despite the flurry of papers that attempt to resolve this conflict by adding new physics to the early universe, we will show that it can instead be resolved via a more careful usage of statistics. Indeed, the Bayesian analyses that produce this conflict are by their nature poorly suited for drawing this type of conclusion. A properly defined frequentist analysis can address this question directly and appears not to find a conflict. Finally, a conservative accounting of systematic uncertainties in measuring the deuterium abundance could reduce what is nominally a 3σ conflict to 1σ .

INTRODUCTION

The recent BOOMERanG [1,2] and MAXIMA-1 [3,4] observations of Cosmic Microwave Background (CMB) anisotropy provide the first high-quality, high-resolution observations to cover the angular scales over which the first two acoustic peaks are expected in the angular power spectrum. If used by themselves, these data are sufficient to determine the location and rough amplitude of the first acoustic peak, providing evidence that the universe is near critical density. A simultaneous fit to numerous cosmological parameters is impossible, however, because of strong degeneracies amongst those parameters in determining the shape of the CMB angular power spectrum at the precision of the observations. As a result, a Bayesian framework has been used in which numerous other cosmological observations are used as priors. These priors come from large-scale structure, Type Ia Supernovae, direct determinations of Hubble's constant, and the baryon density inferred from combining observations of the deuterium-to-hydrogen abundance ratio with the standard predictions of Big Bang Nucleosynthesis (BBN).

A funny thing happened on the way to precision cosmology. While the first acoustic peak is clearly defined by CMB anisotropy data, thereby providing evidence for a flat universe, the second acoustic peak is either at surprisingly low amplitude or missing entirely. Within the parameter space of the standard adiabatic CDM paradigm, this can be produced either by a red-tilted ($n < 1$) primordial power spectrum or by a baryon density higher than that inferred from deuterium observations plus BBN ($\Omega_b h^2 = 0.021 \pm 0.002$) [5]. In order to test the latter idea, [6] performed a Bayesian analysis on the combined BOOMERanG, MAXIMA-1, and COBE-DMR [7] data without using the BBN baryon density as a prior, and they found $\Omega_b h^2 = 0.032 \pm 0.004$. Hence there appears to be a conflict between the CMB and BBN values for the baryon density at roughly the 3σ level.

One way to resolve this conflict is to postulate additional physics in the early universe that alters Big Bang Nucleosynthesis such that the observed deuterium abundance is consistent with the higher value of $\Omega_b h^2$ preferred by the CMB. This has been attempted using degenerate neutrinos due to a large lepton asymmetry [8,9], a decaying neutrino that likewise produces extra entropy during BBN [10,11], or inhomogeneous BBN [12]. Even if the precise priors on the nature of nonstandard BBN are allowed to vary, a robust need for new physics is claimed [13]. Another approach adds new physics to the earlier inflationary epoch in the form of an unexplained bump in the primordial power spectrum of density perturbations [14].

Injecting new physics into the first few minutes of the universe is a serious step and needs to be motivated by a strong observational signal. While the claimed 3σ conflict between CMB and BBN baryon densities seems to have been interpreted by many authors as a sufficient signal, a close examination of the statistics involved reveals that this conflict has been exaggerated and may not exist at all.

BAYESIAN ANALYSES

A Bayesian analysis seeks to answer the question, “Given what I knew before plus the data I have just obtained, what do I now think the truth is?” What was known before is incorporated in the form of a prior probability function. Basic probability theory gives us the starting point,

$$p(model, data) = p(data|model)p(model) = p(model|data)p(data) , \quad (1)$$

which is equivalent to the statement that the probability of two things being true is equal to the probability that the first one is true given that the second one is true times the probability that the second one is indeed true. Bayes’ theorem involves specializing this statement to the case of a set of models and the observed data and dividing by $p(data)$ to get

$$p(model|data) \propto p(data|model)p(model) . \quad (2)$$

The probability that the data would be observed given a particular model is often easy to calculate and is referred to as the likelihood function. This means that as long as we know the prior probability of various models being true, $p(model)$, and can calculate the likelihood function, we can determine the posterior likelihood that each model is correct. We can either think of deuterium observations as part of the current data and do a joint likelihood analysis or we can account for the results of the deuterium observations when we choose a prior probability function for $\Omega_b h^2$. To ignore the deuterium observations entirely would imply that we do not consider them trustworthy.

Since the various models considered by [6] all lie within the adiabatic CDM parameter space, the prior $p(model)$ can be expressed as the product of the prior probability functions of various independent parameters, including the baryon density. When a uniform prior $0.0031 \leq \Omega_b h^2 \leq 0.2$ is used, these authors find a posterior likelihood described by $\Omega_b h^2 = 0.032 \pm 0.004$. If a prior consistent with BBN plus deuterium observations, $\Omega_b h^2 = 0.019 \pm 0.002$ [15], is used, they find a posterior likelihood described by $\Omega_b h^2 = 0.021 \pm 0.003$.

Although these authors conclude that the BBN plus deuterium value of the baryon density is “disfavored by the data,” this is not the right interpretation to ascribe to the results of their Bayesian analysis. If they assume that the BBN plus deuterium value of the baryon density is correct by including it as a prior, they produce a posterior likelihood in good agreement, showing that while the CMB data may favor a higher value of $\Omega_b h^2$, BBN is a much stronger constraint. Indeed, they note that this prior is strong enough to alter the results on other parameters, for instance yielding a scalar spectral index of the primordial power spectrum of $n_s = 0.89 \pm 0.06$ rather than the $n_s = 1.03 \pm 0.08$ produced by the uniform prior on the baryon density. Starting and ending with a baryon density consistent with BBN plus deuterium is a self-consistent result.

When they instead use a uniform prior on the baryon density and ignore the implications of deuterium observations, these authors are starting from the assumption that the BBN value of the baryon density is not worthy of consideration. Producing a posterior likelihood for $\Omega_b h^2$ that is different from the BBN value is again self-consistent; in this case we start and end with the idea that $\Omega_b h^2$ may well be greater than 0.019. In the case of the uniform prior, the CMB data do show the ability to narrow a broad prior into a localized posterior. The correct conclusion to draw from this exercise is that it is quite important to decide a priori whether we believe the deuterium observations and what they imply for the baryon density, because it makes a significant difference in our posterior estimation of the truth.

One complication of Bayesian statistics is that if we do not know the correct priors we should vary our priors over the range of reasonable functions. If indeed both the BBN and the uniform prior on the baryon density are reason-

able, then the correct conclusion about the posterior likelihood is that $\Omega_b h^2$ could be anywhere from 0.02 to 0.03. Because it requires a specific choice of prior assumptions and produces only relative likelihoods at the end, the simple Bayesian analysis is not well-suited to answering the question, “Are the CMB and BBN values for the baryon density in conflict?” This question could be pursued in a Bayesian format using prior assumptions on how likely such a conflict is, but this is far beyond the scope of the analyses that have been done.

FREQUENTIST ANALYSES

A frequentist analysis is a bit simpler to describe; each model is viewed as a separate hypothesis to be tested against the observations. One looks only at the likelihood function i.e. $p(\text{data}|\text{model})$. Typically a misfit statistic such as χ^2 is used, and any models for which the chance of getting a better agreement with the data is greater than or equal to e.g. 95% are considered to be ruled out at the e.g. 95% confidence level. This is more akin to the basic scientific method taught to children; a frequentist analysis seeks to answer the question, “Which of these models are reasonably likely to produce the observed data?” While a best-fit model can still be found, one concentrates on discarding those models that are ruled out beyond some confidence level. Further discrimination requires better data. This frequentist approach has the added benefit of being able to rule out an entire parameter space if none of its models are a reasonably good fit to the data; in this case the hypothesis that the true model lies somewhere in this parameter space has been rejected. The Bayesian approach can be modified to compare one parameter space to another but it always makes a conclusion based on relative likelihood.

In the case of the recent CMB data, a frequentist goodness-of-fit analysis was performed by the MAXIMA team [4]. They find that the Λ CDM model with $\Omega_b h^2 = 0.021$ has $\chi^2 = 10/10$ when compared with the MAXIMA-1 data alone and $\chi^2 = 40/40$ compared with MAXIMA-1 plus COBE-DMR, but their best-fit model with $\Omega_b h^2 = 0.025$ has $\chi^2 = 8/10$ and $\chi^2 = 38/40$ respectively. This is consistent with the Bayesian result that a high baryon density has greater likelihood, but now we have a chance to assess the absolute goodness-of-fit of these models. The “best-fit” model is a slightly better fit than one expects but this is likely explained by having varied seven parameters to find it. Indeed, we should subtract up to seven degrees of freedom from the above results if these seven parameters successfully span the space of possible functions of seventh order. A simple way to state this effect is that even if Λ CDM is the true model we expect that by varying n of its parameters freely we will be able to drop the χ^2 by a value of n ; these parameter variations are fitting the observational errors in the data rather than telling us more about the truth.

Of course, we would like to have a similar frequentist analysis of the full set of CMB data, particularly BOOMERanG, MAXIMA-1, and COBE-DMR. [16] analyze the combined BOOMERanG and MAXIMA-1 data using a relative likelihood analysis of their χ^2 values; since their best-fit model is close to $\chi^2/d.o.f. = 1$ this is nearly the correct frequentist approach. The problem is that they ignore the significant calibration uncertainties of BOOMERanG and MAXIMA-1 so this analysis is seriously flawed and will eliminate a large set of viable models.

An acceptable frequentist analysis of BOOMERanG, MAXIMA-1, and COBE-DMR data has been performed by [14] in the course of adding a bump to the primordial power spectrum in the Λ CDM model. The standard Λ CDM model, i.e. amplitude of bump equals zero, is ruled out at the 68% confidence level but not at 95% confidence. This means that Λ CDM is a reasonably good fit to the current set of high-quality CMB data, and it seems to eliminate the motivation for considering a primordial bump. These authors do not analyze models with higher baryon fractions but most likely would find an even better fit. One must then consider whether a model can be ruled out not for being a bad fit but simply because another model is a better fit. In general, this is a dangerous approach although it is implicit in the Bayesian formalism. If there is evidence to suggest that the observational errors have been overestimated¹ then the relative likelihood approach may be justified, but otherwise it is premature to discard models for which the data is a quite reasonable result.

SYSTEMATIC UNCERTAINTIES IN BARYON DENSITY FROM DEUTERIUM

An alternative manner in which the careful usage of statistics may resolve the apparent conflict between CMB and BBN values of the baryon density is via a fuller accounting of systematic uncertainties in the usage of the observed deuterium-to-hydrogen abundance ratio to infer the baryon density. This issue is explored by [17], who find that while the best-fit CMB baryon density of $\Omega_b h^2 = 0.03$ “cannot be accomodated,” a very conservative consideration of systematic errors would allow $0.016 \leq \Omega_b h^2 \leq 0.025$. Although it is unlikely that the systematic errors in converting the observed deuterium-to-hydrogen abundance ratio into a baryon density are nearly this large, this does allow for the possibility that improved deuterium observations could reduce the claimed 3σ conflict between the CMB- and BBN-preferred baryon densities to 1σ .

Although the above range is quite conservative, the most recent high-redshift quasar absorption system in which [5] measured the deuterium-to-hydrogen abundance ratio, HS0105+1619, would by itself give a result of

¹⁾ How often does this happen in observational astrophysics?

$\Omega_b h^2 = 0.023$. There are reasons to believe that this is the best measurement of deuterium yet performed; it has the highest hydrogen column density and therefore deuterium was seen in several Lyman-series transitions with a reduced chance of contamination from the Lyman alpha forest. Such contamination would increase the perceived abundance of deuterium, leading to an underestimate of the true baryon density. Indeed the HS0105+1619 deuterium-to-hydrogen abundance ratio is higher than the two previous detections of [15] by an amount greater than the observational errors. [5] are forced to add an empirical uncertainty to these points in order to account for their scatter. Unfortunately, it is also possible that mild levels of deuterium destruction due to star formation in the higher metallicity system HS0105+1619 have caused a systematic error in this system instead of the previous ones. Although deuterium destruction is not expected to be significant at the significantly sub-solar metallicity of this system, it is unclear where the true deuterium-to-hydrogen abundance ratio lies amongst the range of observed values.

CONCLUSION

There are thus a number of ways in which a careful usage of statistics seems to eliminate the claimed 3σ conflict between the CMB- and BBN-preferred values of the baryon density. The first is that the Bayesian analyses used are actually producing consistent results; the proper conclusion to be drawn is that whether or not to include prior information from BBN is an important choice. Utilizing relative likelihood information and prior probability functions makes these analyses poorly suited to answering the question of whether a conflict exists between the CMB and BBN values for $\Omega_b h^2$. When a better-suited frequentist analysis is used, we find that the standard Λ CDM model with a BBN plus deuterium preferred value of $\Omega_b h^2 = 0.021$ is in reasonably good agreement with recent CMB observations. While a model with higher baryon fraction may be an even better fit this could simply be caused by having fit several free parameters; we need more precise observations to make a clear discrimination between these models. Additionally it is possible that systematic errors in measuring the deuterium-to-hydrogen abundance ratio are responsible for underestimating the BBN value of the baryon density. Given any one of these reasons, there is no longer a conflict between CMB anisotropy results and the value of $\Omega_b h^2$ preferred by observations of deuterium. Given all of them, it is clearly unnecessary to introduce additional physics to the early universe.

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